

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 1901

A METHOD FOR PREDICTING THE STABILITY
IN ROLL OF AUTOMATICALLY CONTROLLED AIRCRAFT BASED ON
THE EXPERIMENTAL DETERMINATION OF THE CHARACTERISTICS
OF AN AUTOMATIC PILOT

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SUMMARY

A method is suggested for predicting the stability of automatically controlled aircraft by a comparison of calculated frequency-response curves for the aircraft and experimentally determined frequency-response curves for the automatic pilot. The method is applied only to stabilization in roll. The method is expected to be useful as a means of establishing the specifications of the performance required of the automatic control device for pilotless aircraft designed as missiles.

INTRODUCTION

Experience has shown that the provision of automatic stabilization for small pilotless aircraft designed as missiles is extremely difficult. The difficulty is a result of the high-frequency oscillations of small-size aircraft that require rapid control movements and small time lags, characteristics which are difficult to obtain, particularly when the space available for the control servomotors and intelligence units is considered. In an unpublished analysis made at the Langley Aeronautical Laboratory of the NACA, the problem of determining the stability of an automatically controlled aircraft with lag in the control system was analyzed theoretically by assuming a simplified equation for the control motion, this equation being obtained from the knowledge of the behavior of the automatic pilot. Because of the irregular response characteristics often found in automatic pilots, however, the control motion is difficult to represent mathematically and, hence, the simplified equations of the control were found to be inadequate for the analysis.

The present paper suggests a method for predicting the stability of an aircraft based on the experimental determination of the characteristics of its automatic pilot. The procedure consists essentially in calculating the control motion required to maintain a continuous sinusoidal motion of unit amplitude for the degree of freedom being inspected. The motion of the control is obtained for a range of frequencies; the phase angle of the control motion and the ratio of amplitude of control motion to airplane motion are plotted as a function of frequency. Similar curves are established for the autopilot by oscillating it and recording the control motion. The two sets of data are then compared to determine whether the airplane will be stable under control of the automatic pilot. The method is developed in detail only for stabilization in roll. It may be used by the airplane designer for either determining the suitability of an existing automatic pilot for a particular application or specifying the characteristics of the automatic pilot needed for the application.

SYMBOLS

m	mass of airplane, slugs
k_x	radius of gyration of airplane about longitudinal axis, feet
q	dynamic pressure, pounds per square foot
S	wing area, square feet
b	wing span, feet
C_l	rolling-moment coefficient (Rolling moment/ qSb)
ϕ	angle of bank, radians
p	angular velocity in bank, radians per second ($d\phi/dt$)
δ	deflection of aileron, radians
C_{l_p}	rate of change of rolling-moment coefficient with angular velocity in bank, per radian ($\partial C_l / \partial p$)
C_{l_δ}	rate of change of C_l with δ , per radian ($\partial C_l / \partial \delta$)
D	differential operator (d/dt)
ω	angular frequency, radians per second

- θ phase angle (positive value means lead of δ ahead of ϕ)
- ϕ_{\max} maximum amplitude of ϕ
- K control-amplitude ratio (ratio of control deflection to airplane displacement)
- l lag in seconds between signal for control and its actual motion
- t time, seconds
- μ real part of root of stability equation
- $T_{1/2}$ time for oscillation to damp to one-half its amplitude, seconds
- $\delta(t)$ control motion as a function of time
- T period of oscillation, seconds

DETERMINATION OF CONDITIONS FOR NEUTRAL STABILITY

The method of determining the conditions for neutral stability is illustrated in figure 1. The calculated phase angle of the control motion and the calculated ratio of the amplitude of control motion to airplane motion are plotted against angular frequency as shown by the solid-line curves. The upper dashed curve is a plot of the experimental ratio of the amplitude of control motion to autopilot motion against angular frequency. The lower three dashed curves are three possible experimental phase-angle curves for the automatic pilot. The intersection of the experimental and calculated control-amplitude curves establishes the approximate frequency of the airplane with the autopilot in operation. If, as in the case of the intermediate experimental phase-angle curve, the intersection of the experimental and calculated phase-angle curves is at the same frequency as the intersection of the control-amplitude-ratio curves, the airplane may be neutrally stable and may be expected to oscillate continuously at this frequency. It is, however, more usual that the intersection of the experimental and calculated phase-angle curves will not be at the same frequency as the intersection of the control-amplitude-ratio curves. If the phase-angle curves, as in one case shown, intersect at a higher frequency than the control-amplitude-ratio curves, the aircraft will be stable. If, as in the remaining case, the intersection of the phase-angle curves is at a lower frequency than the control-amplitude-ratio curves, the aircraft will be unstable. Because of the nonlinear characteristics of the control system, it is generally necessary to

make the experiments for different amplitudes. With a dead spot (insensitivity to small deviations) there will probably be some amplitude below which the system will be unstable.

Calculated frequency-response curves for the aircraft.— In the application of the method to the case of aileron control of an aircraft or a missile independently stabilized about all three axes, the equation of motion for determining the control movement is

$$\left(\frac{mk_X^2}{qbS} D^2 + C_{Lp} D \right) \phi = C_{L\delta} \delta$$

The calculated steady-state solution of the aircraft in response to a sinusoidal forcing function of unit amplitude $\delta = \sin \omega t$ is $\phi = \phi_{\max} \sin(\omega t + \theta)$. (See reference 1.) The values of ϕ_{\max} and θ are obtained over the desired range of angular frequencies ω by the substitution of $i\omega$ for D in the equation

$$\frac{\delta}{\phi} = \frac{\frac{mk_X^2}{qbS} D^2 + C_{Lp} D}{C_{L\delta}}$$

This substitution is equivalent to specifying an undamped sinusoidal motion and results in the expression $A + iB$ from which can be

obtained $\frac{1}{\phi_{\max}} = \sqrt{A^2 + B^2}$ and $\theta = \tan^{-1} \frac{B}{A}$. The angle θ may denote either a phase lag or lead, depending upon its quadrant. If θ is in the third or fourth quadrant, the control lags behind the displacement, but if θ is in the first or second quadrant, the control leads the motion of the airplane. The ratio of the amplitudes δ and ϕ is $\frac{1}{\phi_{\max}}$ and may be defined as the control-amplitude ratio K of the control system, that is, the ratio of maximum control deflection to maximum displacement in bank. A plot of K and θ against ω shows the combination of control-amplitude ratio and phase lag or lead necessary to maintain fixed-amplitude oscillations at any given frequency. These results are the calculated frequency-response curves due to a sinusoidal motion of the aircraft.

Determination of equivalent sine wave for the automatic-pilot response.— The experimental frequency-response curves are obtained by oscillating the automatic pilot sinusoidally at various amplitudes through the desired range of frequencies. The control is assumed to oscillate at the same frequency as the automatic pilot but because of

the physical characteristics of the autopilot the control motion may differ widely from a true sine wave and may show arbitrary phase, amplitude, or wave-form relations (fig. 2). It is necessary, therefore, to determine an equivalent sine-wave response for any arbitrary control motion.

In order to determine the equivalent sine wave for an arbitrary control motion, the following relations are assumed:

(1) The work done per cycle by a control following the equivalent sine wave on an aircraft having a harmonic displacement $\sin \omega t$ must be the same as that done by the actual control variation.

(2) The angular impulse of the equivalent sine wave acting on the airplane over a half cycle must equal the change in angular momentum of the airplane caused by the actual control motion during the same interval.

The work done by a nonharmonic force $\delta(t)$ of frequency ω upon a harmonic motion $\sin \omega t$ is proportional to

$$B_1 = \frac{2}{T} \int_0^T \delta(t) \cos \omega t \, dt$$

where the period of the oscillation is $T = \frac{2\pi}{\omega}$, and B_1 is the coefficient of the component $\cos \omega t$. (See reference 2.) This component of the control motion that is out of phase with the aircraft motion is the only harmonic of the Fourier series representing the forcing function $\delta(t)$ which contributes to the work done on the aircraft. The angular impulse is obtained by integrating the curve of control deflection against time over a half cycle. This component of the control motion in phase with the sinusoidal motion of the aircraft, obtained from the second relation, is

$$A_1 = \frac{\pi}{T} \int_0^{T/2} \delta(t) \, dt$$

where A_1 is the coefficient of the component $\sin \omega t$. The condition of zero net impulse over a series of cycles may be met by adjusting the

reference axis for $\delta(t)$ so that $\int_0^T \delta(t) \, dt = 0$. The control motion

may then be expressed as the sum of the in-phase and out-of-phase components:

$$A_1 \sin \omega t + B_1 \cos \omega t$$

or

$$\sqrt{A_1^2 + B_1^2} \sin (\omega t + \theta)$$

The control-amplitude ratio K is equal to $\sqrt{A_1^2 + B_1^2}$ and the phase lag of the system θ is $\tan^{-1} \frac{B_1}{A_1}$. The control-amplitude ratio and the phase lag or lead are determined from records taken of the oscillations and plotted against ω . In contrast to the calculated frequency-response curves which involve the aerodynamic and mass characteristics of the aircraft, these experimentally determined curves will be functions of the dead spots and various types of lag found in the control system. In general, the behavior of the automatic pilot will be nonlinear and hence a family of curves showing different phase and amplitude relations for different amplitudes of disturbance will be obtained.

Comparison of the calculated frequency-response curves of the aircraft and the experimental frequency-response curves of the automatic pilot.— The two sets of frequency-response curves show, on the one hand, the values of K and θ necessary for hunting at a given frequency and, on the other hand, the actual values of K and θ obtained experimentally at this frequency. In order to determine from these curves whether the aircraft will hunt in flight, the following conditions must be satisfied:

- (1) At a given frequency and amplitude the experimental values of K and θ must agree with the calculated values.
- (2) The motion must be stable for amplitudes larger than the one at which the airplane will hunt (as determined from the first condition).

The first condition indicates that the control-amplitude ratio and phase lag or lead obtained as a result of all types of lag in the control system must agree with the combination of K and θ necessary for hunting to exist. The second condition is essential to prevent instability if the aircraft is displaced to amplitudes larger than the one at which it will hunt. The aircraft is stable at these larger amplitudes if, at the frequency for which the calculated and experimental control-amplitude ratios are equal, the calculated value of the phase lag required for hunting is greater than the experimental value.

In other words, the calculated value of θ is a critical value of the lag necessary to cause the aircraft to hunt. If experimental values of θ are less than the critical value, the aircraft motion is damped, whereas instability occurs if the experimental value of θ exceeds the calculated critical value (fig. 1).

Illustrative case.— The equation of motion in bank of a small experimental aircraft tested in the Langley 7- by 10-foot tunnel was estimated as

$$(0.000245D^2 + 0.00245D)\phi = -0.2645\delta$$

Solving the equation for δ/ϕ and substituting $i\omega$ for D give the expression

$$\frac{\delta}{\phi} = 0.000926\omega^2 - 0.00926\omega i$$

The resultant values of K and θ , that is, the calculated frequency-response curves for the aircraft, are shown as solid lines in figure 3. These curves show that for small values of K , the frequency of the steady oscillation is low and the motion will not be sustained unless the phase lag is large. As K increases, the frequency of the steady oscillation increases but the phase lag required decreases. It is important to note that an automatic pilot with a constant time lag ℓ would be unstable at high angular frequencies since the relation between angular frequency, phase lag, and time lag is θ (radians) = $\omega\ell$.

The experimental frequency-response curves were obtained by oscillating the automatic pilot at amplitudes of 10° and 20° through the range of desired angular frequencies. The phase lag and control-amplitude ratio for the two amplitudes were determined from records similar to figure 2 and are plotted as a function of ω in figure 3 for two values of control-amplitude ratio K . For this particular automatic pilot, the control-amplitude ratio was independent of amplitude whereas the phase lag varied with amplitude. The results in figure 3 indicate that, for each control-amplitude ratio, the experimental values of θ are greater than the calculated phase lag, and hence the aircraft would be unstable. Unpublished results from wind-tunnel tests indicated that the motion was unstable, as predicted from the curves of figure 3.

In an effort to make the aircraft stable, the parameters of the automatic pilot were modified and additional wind-tunnel tests were performed. The conditions selected for these wind-tunnel tests were, however, different from those conditions for which the experimental frequency-response curves were obtained and hence no direct prediction of the aircraft stability could be made. The results of the wind-tunnel tests with the modified automatic pilot indicated a steady

oscillation, and records taken of the tests showed that the values of K and θ agreed very closely with the combination of K and θ determined from the calculated frequency-response curves. The test points of figure 4 show the combination of K and θ for the cases in which steady oscillations occurred in the roll tests.

CALCULATED FREQUENCY RESPONSE OF THE AIRCRAFT FOR DAMPED OSCILLATIONS

The previous calculations of the frequency-response curves were based on the assumption that the sinusoidal motion of the aircraft is neutrally damped. It is often desirable, however, to determine the performance of an automatic control device required to cause the motion of the aircraft to damp at a sufficiently rapid rate. Although no satisfactory analysis of this problem has been given, a qualitative indication of the rate of damping to be expected in a given case may be obtained by comparing the measured phase and amplitude of the control to the phase and amplitude calculated to be required to enforce a given rate of damping.

Strictly speaking, the assumed exponential damping of the motion would require an exponential decrease in the response of the autopilot at decreasing amplitudes. In general, such a linear response cannot be expected and hence the method will require careful judgment in its application.

The equation of damped motion in bank for the illustrative case may be written by adding a real part μ to the imaginary root $i\omega$

$$[0.000245(-\mu + i\omega)^2 + 0.00245(-\mu + i\omega)]\phi = -0.2645\delta$$

where μ is given a value as determined by the desired rate of damping. The motion damps to one-half its amplitude in $T_{1/2} = \frac{0.693}{\mu}$ seconds. Solving the equation for δ/ϕ gives the expression

$$\frac{\delta}{\phi} = 0.000926\omega^2 - 0.000926\mu^2 + 0.00926\mu + i(-0.00926\omega + 0.001852\omega\mu)$$

The frequency-response curves shown in figure 5 were calculated for values of μ varying from 0.175 to 8.31. The control-amplitude-ratio curves are only plotted for values of μ equal to 0 and 5 since they are the limiting curves for the values of μ investigated. Figure 5 indicates that the control-amplitude ratio is almost independent of μ

whereas the phase lag decreases as μ increases. For a control system with a given control-amplitude ratio, therefore, the damping of the oscillation increases as the phase lag is reduced. If the oscillation is to damp one-half in less than 1/7.22 second, the control motion must lead the aircraft motion.

In order to predict quantitatively the stability of the motion of an aircraft which damps exponentially, the experimental frequency-response curves would have to be obtained for the condition where the forced oscillation of the automatic pilot also damps exponentially.

CONCLUDING REMARKS

A method for predicting the stability in roll of automatically controlled aircraft by a comparison of calculated frequency-response curves for the aircraft and experimentally determined frequency-response curves for the automatic pilot is presented. The method is expected to be useful as a means of establishing the specifications of the performance required of the automatic control device for pilotless aircraft designed as missiles.

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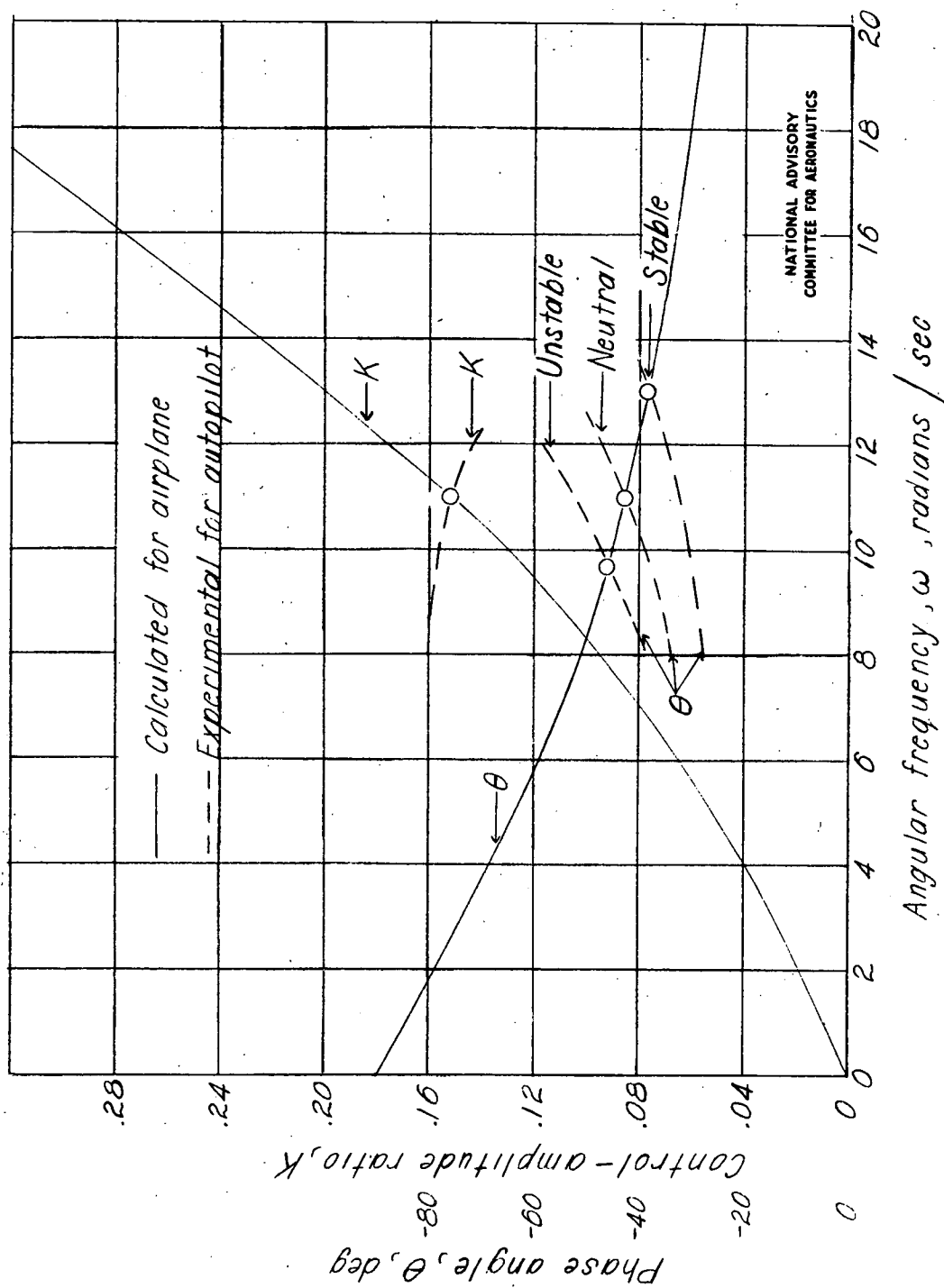


Figure 1.— Frequency-response curves for stable, unstable, and neutrally damped motion of the aircraft.

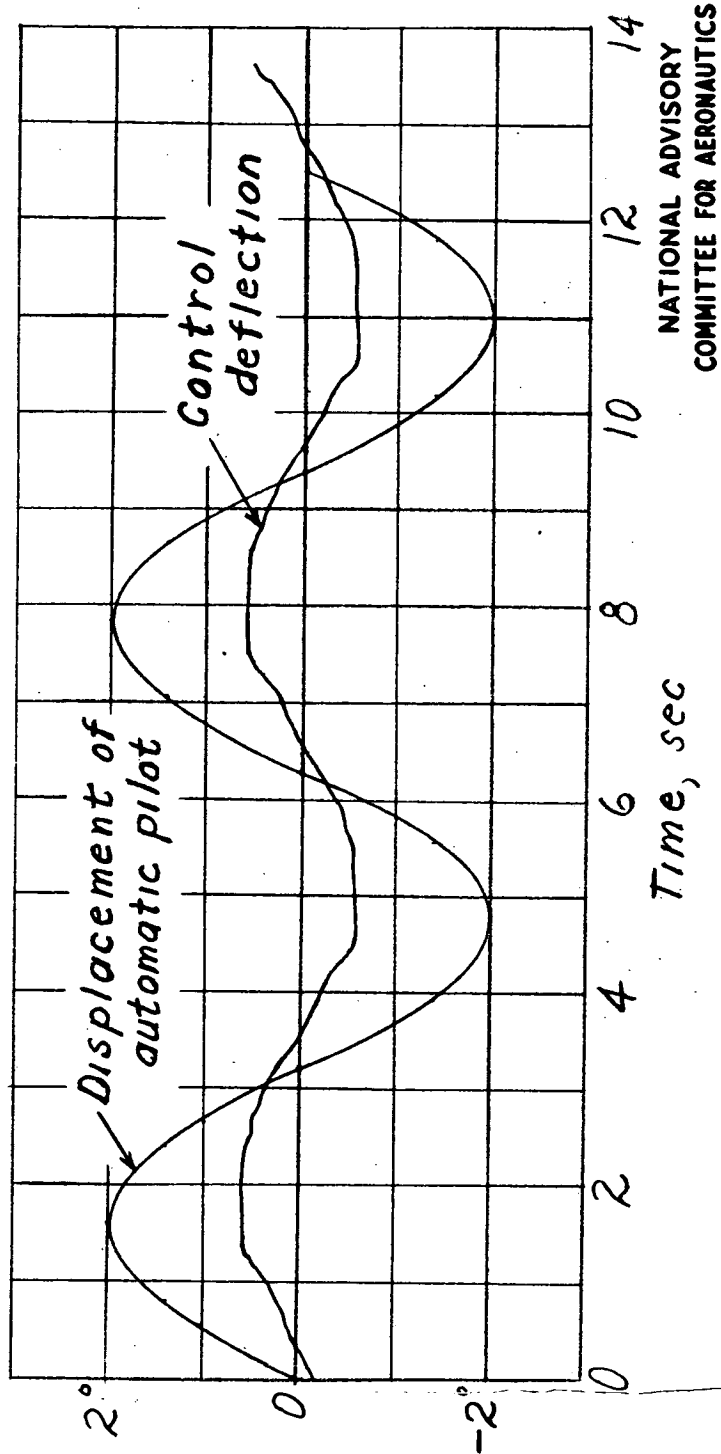


Figure 2.— Typical response of control to sinusoidal oscillation of automatic pilot.

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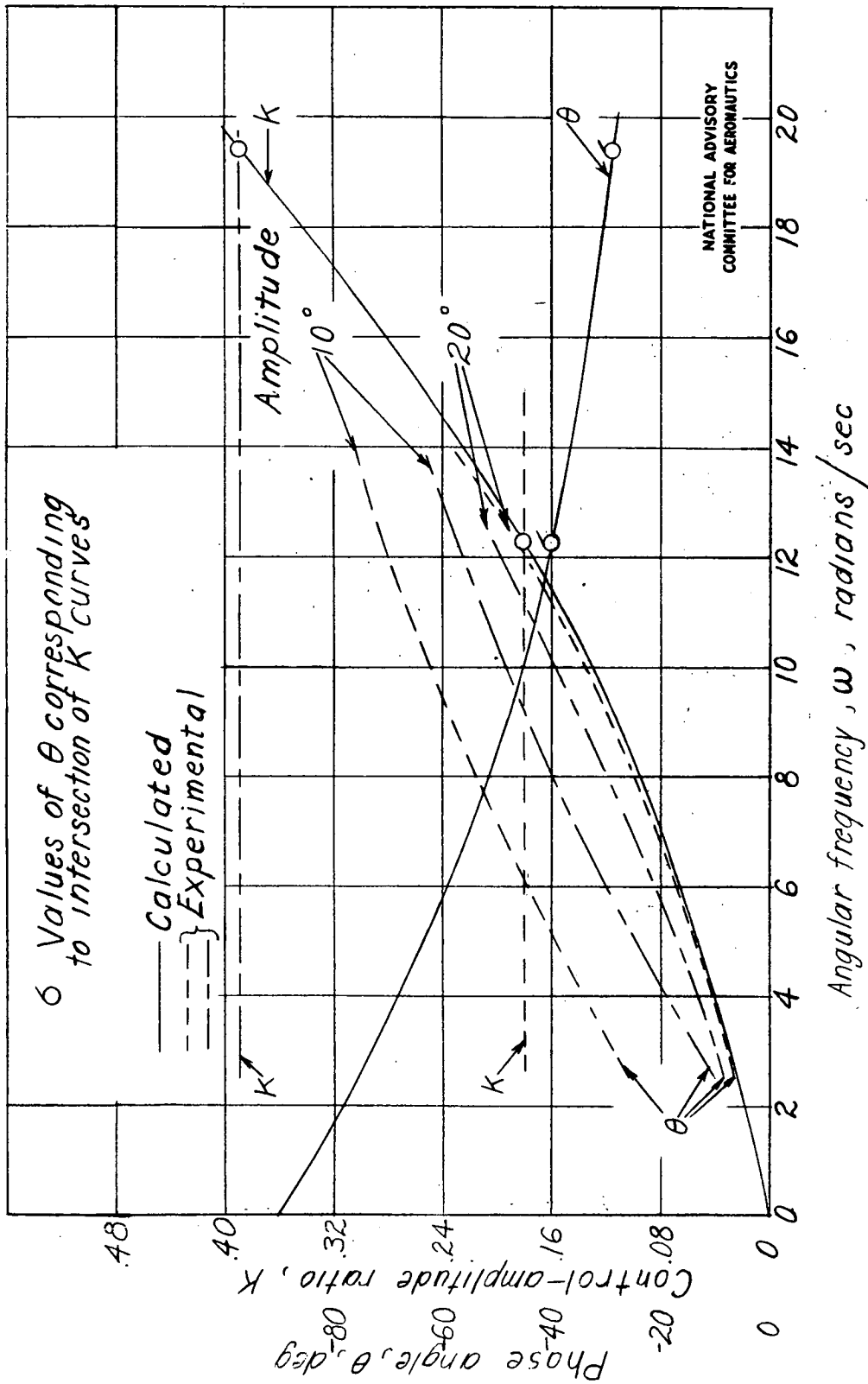


Figure 3.— A comparison of the calculated and experimental frequency-response curves of a small aircraft.

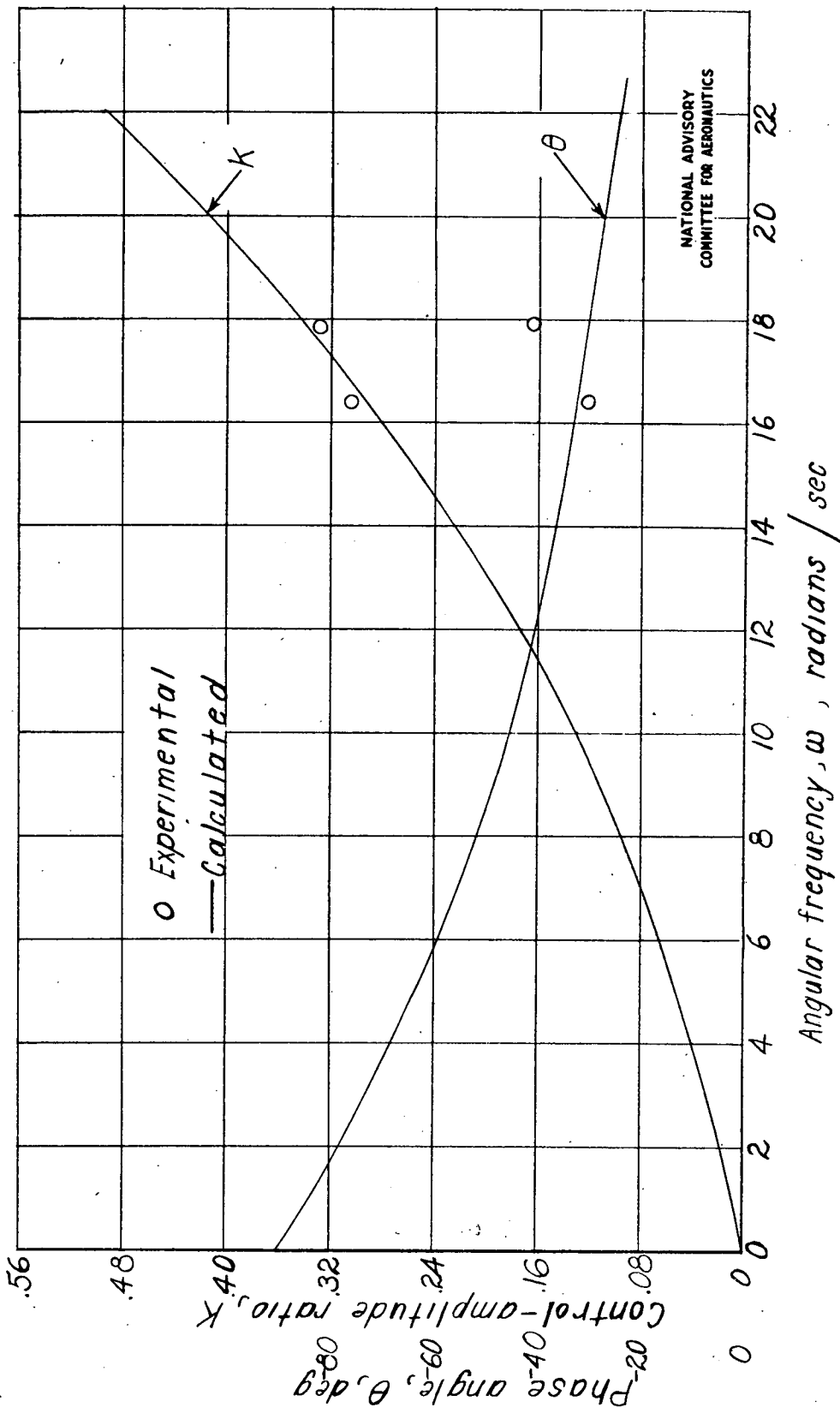


Figure 4.— A comparison between roll-test results and calculated frequency-response curves for a small aircraft.

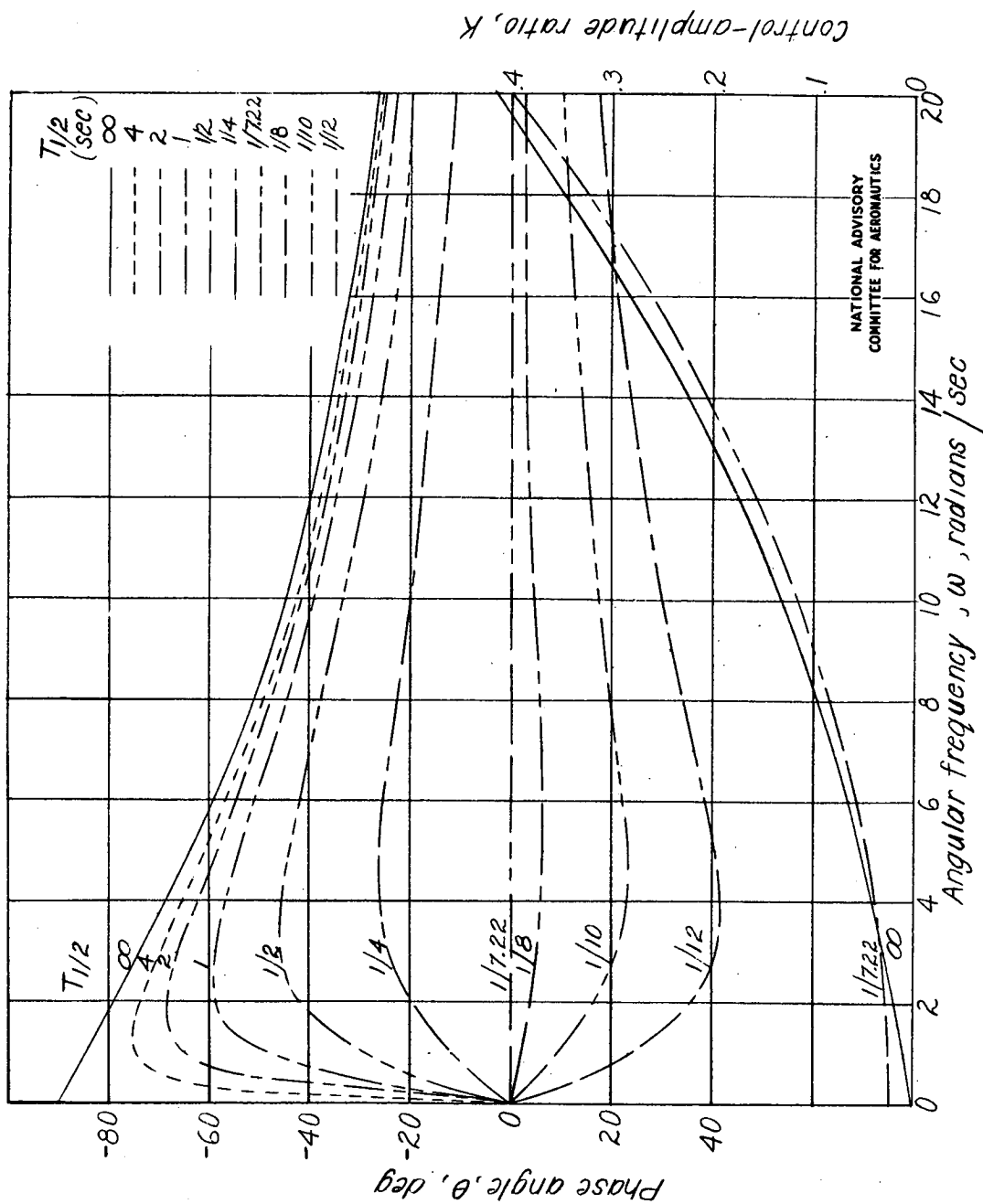


Figure 5.— Calculated frequency-response curves of the aircraft for damped oscillations.